Quantum Information and Quantum Computing, Project 2

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A quantum algorithm for the steady state of an open quantum system

An rather unkown researcher has recently posted on arxiv.org a manuscript, where a quantum algorithm is developed to directly estimate the non-equilibrium steady state (NESS) of an open quantum system.

An open quantum system interacting with a Markovian environment evolves in time according to the Lindblad-Von Neumann equation for the density matrix. This model very often predicts a NESS, whereby the density matrix evolves irreversibly in time and reaches asymptotically a steady state. Knowing the NESS is very important when modeling open quantum systems.

Several quantum algorithms have been recently proposed to compute the time-evolution governed by this equation, using a quantum computer. However, the time needed to reach the asymptotic steady state may often be so long that any simulation strategy based on time evolution may be inefficient. The work proposed in the present project adopts a different approach. It directly computes the NESS by solving the algebraic equation resulting from the Lindblad-Von Neumann equation when setting the time derivative to zero. For this, it takes advantage of the quantum phase estimation algorithm and of an efficient preparation of the initial state. For systems where the Liouvillian gap (see the article for detail) doesn't scale down exponentially with the system size, the quantum algorithm provides an exponential speedup over the exact diagonalization on a classical computer.

The goal of the project is:

1. Read and understand the main article and present its results.
2. Implement on IBM-Q the smallest instance of the algorithm (which should require 5 qubits), for the simulation of the NESS of a single spin subject to spin relaxation and an external field. The equations are
$$\frac{d\hat{\rho}}{dt} = -i[\hat{H}, \hat{\rho}] - \frac{1}{2}\hat{\sigma}^+\hat{\sigma}^-\hat{\rho} - \frac{1}{2}\hat{\rho}\hat{\sigma}^+\hat{\sigma}^- + \hat{\sigma}^-\hat{\rho}\hat{\sigma}^+$$
$$\hat{H} = h\hat{\sigma}_x + h'\hat{\sigma}_y$$
where h and h' are the x and y components of an external field acting on the spin, and $\hat{\sigma}^{\pm} = \hat{\sigma}_x \pm i\hat{\sigma}_y$.
3. Test the algorithm by estimating some simple observables, with the method presented in the last part of the manuscript. Compare with the exact result computed numerically.
4. Run the circuit with the QASM simulator including noise and study how rapidly the quality of the solution deteriorates as a function of the circuit depth.
5. (optional) Argue on how the noise issue found at the previous step could be overcome by optimizing the algorithm.